

Thermal effects on the absorption of ultra-high energy neutrinos by the cosmic neutrino background

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Abstract. We use the formalism of finite-temperature field theory to study the interactions of ultra-high energy (UHE) cosmic neutrinos with the background of relic neutrinos and to derive general expressions for the UHE neutrino transmission probability. This approach allows us to take into account the thermal effects introduced by the momentum distribution of the relic neutrinos. We compare our results with the approximate expressions existing in the literature and discuss the influence of thermal effects on the absorption dips in the context of favoured neutrino mass schemes, as well as in the case of clustered relic neutrinos.

1. Introduction

The interaction of cosmic neutrinos at ultra-high energies (UHE ν) with the cosmological background of relic (anti) neutrinos (C ν B) has been proposed as an indirect way of observing the C ν B [1]. Provided adequate sensitivity and energy resolution of the detectors, the observation in the UHE ν flux of absorption lines associated with the resonant production of a Z boson ($\nu\bar{\nu} \rightarrow Z \rightarrow f\bar{f}$) could allow the determination of the absolute neutrino masses [2]. The shape and depth of these absorption dips may also reflect features of the distribution of UHE ν sources and their emission spectrum [3, 4]. Most of the work in the literature describe the UHE ν -C ν B interactions assuming that relic neutrinos are at rest. The effects of thermal motion in the C ν B (whose present temperature is $\approx 1.69 \times 10^{-4}$ eV), however, become relevant as soon as the momentum of the relic neutrinos gets comparable to their mass. In this work, we take into account the thermal effects on the dominant (resonant) contribution to the neutrino damping by using finite-temperature field theory (FTFT) [5]. In section 2 we present the UHE ν transmission probability across the C ν B for various mass schemes, and in section 3 we briefly discuss the impact of thermal motion on the absorption in relic neutrino clusters.

2. Transmission probability of UHE ν across the C ν B

For an UHE ν with four-momentum $k^\mu = (\mathcal{E}_K, \vec{K})$ and mass m_ν travelling across the C ν B, the equation of motion reads $(\not{k} - m_\nu - \Sigma) \psi = 0$. The self-energy Σ embodies the effects of the surrounding neutrino medium, coming in this case from the exchange of a Z boson. The corresponding dispersion relation is given by $\mathcal{E}_K = \mathcal{E}_r - i\frac{\gamma}{2}$, where \mathcal{E}_r and γ are functions of K . In the real-time formalism of FTFT, one calculates Σ from the corresponding Feynman diagram, having in mind that the vertices are doubled respect to the standard theory, and that the propagators of the Z boson (near the resonance) and of the relic neutrino become 2×2 matrices. The relic neutrino propagator also depend on the functions $f_\nu(P)$ and $f_{\bar{\nu}}(P)$ characterizing the

momentum distribution of (anti) neutrinos in the thermal bath. These functions take the simple relativistic Fermi-Dirac form $f_\nu(P) = f_{\bar{\nu}}(P) = 1/(e^{P/T_\nu} + 1)$, where T_ν is the temperature of the CνB and we have neglected the chemical potential.

The damping factor γ , which governs the attenuation of the UHEν flux across the background of relic neutrinos, is directly related to the imaginary part of the self-energy Σ_i [6]. In the approximation that the UHEν are ultrarelativistic and that we can neglect the background effects on their energy ($\mathcal{E}_r \simeq K$), we may write the damping as (see [5] for the detailed calculation)

$$\gamma_{\nu\bar{\nu}}(K) = -\frac{1}{K} \text{Tr}(k\Sigma_i)|_{\mathcal{E}_r=K} = \int_0^\infty \frac{dP}{2\pi^2} P^2 f_{\bar{\nu}}(P) \sigma_{\nu\bar{\nu}}(P, K). \quad (1)$$

For $m_\nu \ll M_Z, K$ and neglecting terms of order Γ_Z^2/M_Z^2 , we have

$$\begin{aligned} \sigma_{\nu\bar{\nu}}(P, K) = & \frac{2\sqrt{2}G_F\Gamma_Z M_Z}{2KE_p} \left\{ 1 + \frac{M_Z^2}{4KP} \ln \left(\frac{4K^2(E_p + P)^2 - 4M_Z^2 K(E_p + P) + M_Z^4}{4K^2(E_p - P)^2 - 4M_Z^2 K(E_p - P) + M_Z^4} \right) \right. \\ & \left. + \frac{M_Z^3}{4K\Gamma_Z} \left[\arctan \left(\frac{2K(E_p + P) - M_Z^2}{\Gamma_Z M_Z} \right) - \arctan \left(\frac{2K(E_p - P) - M_Z^2}{\Gamma_Z M_Z} \right) \right] \right\} \quad (2) \end{aligned}$$

where $E_p = \sqrt{P^2 + m_\nu^2}$ is the energy of the relic neutrino. Taking the limit of eq.(2) for $P \rightarrow 0$, one recovers the approximated cross-section which is used for relic neutrinos at rest, with the Z peak at the "bare" resonance energy $K_{res} = M_Z^2/(2m_\nu)$.

The transmission probability for an UHEν emitted at a redshift z_s to be detected on Earth with an energy K_0 is obtained by integrating the damping along the UHEν path, taking into account that both the UHEν energy and in the CνB temperature are redshifted:

$$P_T(K_0, z_s) = \exp \left[- \int_0^{z_s} \frac{dz}{H(z)(1+z)} \gamma_{\nu\bar{\nu}}(K_0(1+z)) \right], \quad (3)$$

where $H = H_0 \sqrt{0.3(1+z)^3 + 0.7}$ is the Hubble factor. We determined the absorption dips in P_T for m_ν ranging from 10^{-1} eV to 10^{-4} eV (see [5]) and found that the approximation of relic neutrinos at rest breaks down as soon as $m_\nu/T_\nu \lesssim 10^{-2}$: the absorption dips get broadened and shift to lower UHEν energies, and the effect increases with z_s . Fig. 1 shows the average transmission probability \bar{P}_T obtained by summing on the neutrino flavours [3], for mass patterns compatibles with the currently favoured 3-neutrinos mass schemes [7]. The absorption dip corresponding to the smallest mass is almost always washed out and only contributes to further broadening the absorption dip at high energies. At very large z_s , the merging of the two other absorption dips in the normal hierarchy scheme makes it more difficult to differentiate from the inverted hierarchy one. We conclude that thermal effects significantly affect the UHEν transmission probability well before the relic neutrinos become relativistic. This complicates the extraction of m_ν and z_s from the startpoint and endpoint of the absorption dip (which, in the approximation $P = 0$, were located at $K_0 = K_{res}/(1+z_s)$ and $K_0 = K_{res} = M_Z^2/(2m_\nu)$ respectively).

3. Absorption lines due to relic neutrino clustering

According to recent calculations [8], depending on their mass and on the properties of the dark matter distribution, relic neutrinos cluster on scales ranging from 0.01 to 1 Mpc with overdensity factors N_{cl} of the order of $10 - 10^4$ with respect to the present background density ($n_{\nu 0} \approx 56\text{cm}^{-3}$ per species). To compute the UHEν absorption by clustered neutrinos we replaced $f_\nu(P)$ in eq. 1 by a modified Fermi-Dirac distribution,

$$f_\nu^{cl}(P) = \frac{1}{2} \frac{e^{-\Phi/T_\nu} + 1}{e^{(P-\Phi)/T_\nu} + 1}, \quad (4)$$

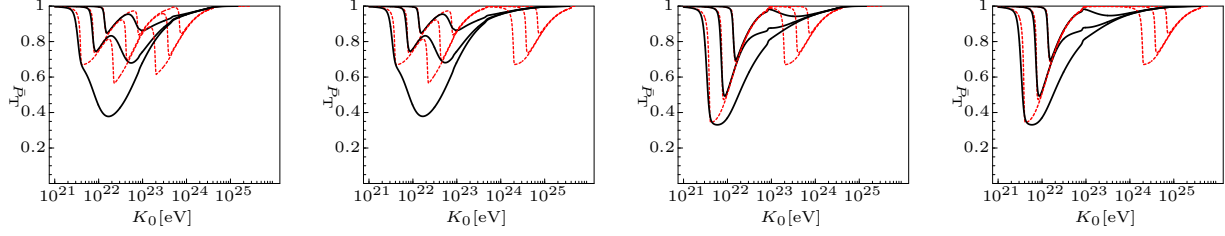


Figure 1. Average transmission probability \bar{P}_T as a function of the UHE ν energy as detected on Earth, K_0 , for a source located at redshifts $z_s = 5, 10, 20$ (from top to bottom in each plot). The two left plots assume a normal hierarchy for the m_ν , with values $(5 \cdot 10^{-2} \text{ eV}, 9 \cdot 10^{-3} \text{ eV}, 10^{-3} \text{ eV})$ and $(5 \cdot 10^{-2} \text{ eV}, 9 \cdot 10^{-3} \text{ eV}, 10^{-4} \text{ eV})$ respectively. The right plots assume an inverted hierarchy with masses $(5 \cdot 10^{-2} \text{ eV}, 5 \cdot 10^{-2} \text{ eV}, 10^{-3} \text{ eV})$ and $(5 \cdot 10^{-2} \text{ eV}, 5 \cdot 10^{-2} \text{ eV}, 10^{-4} \text{ eV})$ respectively. The continued, black curves correspond to the full damping as from eq. (1) and (2), while the dotted (red) curves are for the approximation of relic neutrinos at rest.

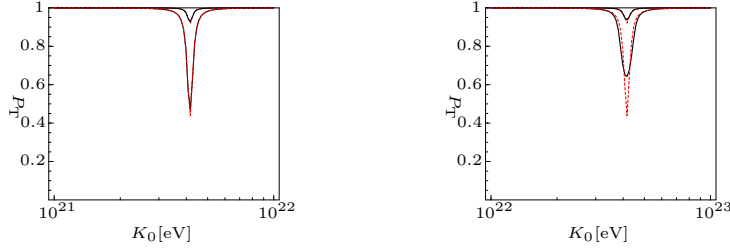


Figure 2. Transmission probability for a cluster of extension 1 Mpc, made of neutrinos of mass 1 eV (left) and 10^{-1} eV (right), with a constant neutrino density $n_\nu^{\text{cl}} = 10^3 n_{0\nu}$ and $n_\nu^{\text{cl}} = 10^4 n_{0\nu}$ (from top to bottom in each plot). The colour code is the same as in fig. 1.

which fits reasonably well the profiles obtained in numerical calculations using Vlasov equation [8]. For a given overdensity factor N_{cl} we solve $\int_0^\infty \frac{dP}{2\pi^2} P^2 f_\nu^{cl}(P) = N_{cl} n_{\nu 0}$ for Φ . We computed the UHE ν transmission probability across a cluster located between the UHE ν source and the observer and found that the effect of the thermal motion of relic neutrinos is in general small or negligible. This is essentially because clustering is efficient only for neutrinos with mass $m_\nu \gtrsim 0.1 \text{ eV}$, and because it is achieved only at small redshifts, when the C ν B temperature is still very small. As shown in fig. 2, we have to saturate the bounds on the parameters to obtain a significant effect. For a maximal overdensity factor $N_{cl} = 10^4$, thermal effects reduce the maximum absorption probability across the cluster from $\approx 55\%$ to $\approx 35\%$.

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